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Heat and mass transfer in a cylindrical grain silo submitted to a periodical wall heat flux

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Abstract—A grain silo is submitted to periodic and meteorologic influences: solar radiation, air temperature, wind. The heat transfer from the wall provokes a natural convective air flow and the grain out of equilibrium releases water. We consider heat and mass transfer in a cylindrical silo submitted to a uniform and periodic wall heat flux. The space under and above the bulk grain in the silo is free. A physical scaling of the problem leads us to build a dimensionless system of equations with three unusual numbers. An experimental study is carried out on a reduced silo filled with moistened mineral grain. The temperature and air moisture fields are investigated with thermocouples and sensors. Integration of equations allows us to evaluate natural convective velocity and quantity of water removed. Two flowing configurations are deduced: during the first half period, when the heat flux is high, the main part of the heat and mass transfer takes place in a boundary layer at the wall and this layer is fed with fresh air from the bottom and from the top through a central aspiration. During the second half period, when the heat flow is low, the stored heat provides a convective flux of a chimney type.

1. INTRODUCTION

THE STORING of granular or powdery products in large containers such as silos is widely used in the agricultural and industrial sectors. To stabilize many of those granular materials, drying them is necessary before storing them. The moisture content fixed for this state guarantees equilibrium with ambient air. For example, corporations buy cereals with a water content below 15% and this criterion is strictly respected to avoid mouldiness which could infect all the stock. The equilibrium curves show water content in relation to temperature and relative moisture of air. An increase in temperature at a point of a silo provokes a decrease of equilibrium water content and as a result a release of water vapour which migrates into the porous medium.

The silos are often very high (10–50 m) in open fields and are exposed to atmospheric conditions: solar radiation, air temperature, wind. The silo walls can offer a partial thermal protection against external conditions. A thick concrete wall is better, from this point of view, than a thin metal one. The choice of the constitutive material of the silo wall results from strength calculations and economic considerations. Many such buildings are made of a light material with

poor insulation. For this kind of silo the grain layer against the wall is submitted to a heat flux, resulting from solar radiation and external air temperature, through the wall. The physical feature of this thermal action is the variability, day–night or seasonal alternance, that we shall schematize as periodic thermal flux.

The heat flux on the side of the bulk grain in the silo provokes air and grain heating, and thus vapour production. An air temperature or moisture gradient creates a natural convection which can transport the moist air further, where it condenses on contact with the colder wall. The managers of storage installations know particular places where liquid water filters on to the grain (top of the column, north face . . .).

Our purpose in this paper is to understand the mechanisms of this particular convection, and to predict the amount of water displaced. The silo geometry selected is cylindrical and consequently we have schematized a two-dimensional cylindrical problem. The bulk grain is generally not confined to the silo volume. Some upper apertures provide a natural ventilation on top of the grain and it is a good thing to limit vapour condensation. We can consider in such a way one open volume for the air surrounding the grain

NOMENCLATURE

<p>C concentration, specific heat at constant pressure</p> <p>g gravitational acceleration</p> <p>\mathbf{G} gravitational vector</p> <p>H height of the silo</p> <p>\mathbf{i} vertical unit vector</p> <p>k mass exchange coefficient</p> <p>K permeability</p> <p>L latent heat</p> <p>N buoyancy ratio</p> <p>P pressure</p> <p>q heat flux at the wall</p> <p>Q thermal ratio</p> <p>r polar coordinate</p> <p>R radius of the silo</p> <p>Ra Rayleigh number</p> <p>Re Reynolds number</p> <p>S section area</p> <p>t time</p> <p>T temperature</p> <p>u vertical velocity component</p> <p>v polar velocity component</p> <p>\mathbf{V} velocity vector, celerity</p> <p>\mathcal{V} celerity ratio</p> <p>x vertical coordinate</p> <p>X grain water content (d.b.)</p> <p>Y absolute air moisture.</p>	<p>β coefficient of volumetric thermal expansion</p> <p>β_c, β_v coefficient of concentration expansion</p> <p>δ thickness of boundary layer</p> <p>ε porosity</p> <p>λ thermal conductivity</p> <p>μ viscosity</p> <p>ν kinematic velocity</p> <p>ξ volumetric exchange surface</p> <p>ρ density</p> <p>τ period</p> <p>ϕ enthalpy flux.</p>
<p style="text-align: center;">Subscripts</p>	
<p>a dry air</p> <p>cond. conduction</p> <p>d.f. drying front</p> <p>eq. equilibrium</p> <p>ex. excitation</p> <p>g gas</p> <p>n.c. natural convection</p> <p>p granular product</p> <p>o outside conditions</p> <p>ref. reference</p> <p>sat. saturation</p> <p>v water vapour</p> <p>w liquid water, wall.</p>	
<p style="text-align: center;">Superscript</p>	
<p>* effective value.</p>	

Greek symbols

α thermal diffusivity

top. The condition at the inferior level is questionable. Some silos are supplied with impervious trap doors. For many of them, the ventilation ducts create air apertures sufficient to assure the possibility of a little flow induced by natural convection. Thus we suppose there is an air open volume under the grain bulk.

Therefore the model of the considered problem is :

- a cylindrical container of grains,
- a periodic flux at the wall,
- open air under and above the bulk grain,
- a thermic and mass natural convection in the porous medium,
- the grain is a moisture source in relation to air temperature and moisture.

2. HEAT AND MASS TRANSFER IN GRANULAR POROUS MEDIA

Natural convection flows in porous media are encountered in many geophysical and engineering applications. Many processes related to low level of energy recovery or environmental protection induce prob-

lems of natural convection in porous media. The number of publications about this subject grows rapidly from year to year. The identification of the research field leads to the following topics :

- heat transfer or mass transfer driven flows,
- confined or not confined porous media,
- transient or permanent convection,
- Darcy or Darcy-Forschheimer-Brinkman model,
- specific boundary conditions (vertical or horizontal heated wall . . .).

This field of natural convection concerns the media saturated with one fluid phase, generally gaseous. Considering low air velocity, the heat and mass transfers are driven by the combined effects due to temperature and density gradients. The flows can be subdivided into heat transfer driven flows ($N \ll 1$) and mass transfer driven flows ($N \gg 1$), where N is the buoyancy ratio [1] :

$$N = \frac{|\beta_c \Delta C|}{|\beta \Delta T|}$$

The intermediate case ($N \sim 1$) is rarely considered in porous media because it is too complex. The mass transfer is generally presented as a diffusive problem from a boundary [1–3].

The porous medium is generally heated through a wall from which natural convection extends. In the case of vertical walls, the boundary layer concept is efficient to obtain solutions. The simplest model studied by Cheng and Minkowycz [4] is the vertical plate in a semi-infinite porous reservoir. The thermal boundary is $\delta_T \sim x(Ra_x)^{-1/2}$, where Ra_x is the Darcy modified Rayleigh number $Ra_x = Kg\beta x\Delta T/\alpha\nu$, x the coordinate along the wall.

Natural convection in confined porous media is the subject of many works [5]. The boundary layer solution is a noteworthy approximation of the exact one. The natural convection in the porous medium open to external air is rarely studied [6]; the coupling with the exterior space brings additional difficulties. Studies on transient convection generally concern the establishment of a state from initial time where temperature or flux is applied on the wall [7].

The momentum equation in porous media is usually 'the Darcy law' initially based on measurements, and later established with an 'area averaged operation' from the Navier–Stokes equation written for the flow in the pore [8]. When the fluid velocity becomes large, i.e. $uK^{1/2}/\nu > 1$, Forscheimer adds a quadratic term ($\sim u^2$) to the Darcy law. When it is necessary to express a null velocity condition on a wall, Brinkman suggests adding another term following the boundary layer equation [9]. Sometimes, a transient term $\rho(\partial V/\partial t)$ is added to the Darcy equation. The scaling study indicates whether it is necessary to take it into account.

Now we select some studies which enlighten this defined problem. Thibaud [10] considers the same geometrical configuration of a silo (cylindrical, open extremities) with a stationary and only natural heat convection provoked by a lateral uniform flux. The numerical study leads to two classes of flows (Fig. 1).

(i) For narrow silos ($R/H < 1$), or small value of Rayleigh number $Kg\beta H^2 q/\alpha\nu$, i.e. a weak wall flux, the flow is of chimney type. The convective draught provokes an air inlet only at the base.

(ii) For large silos ($R/H > 1$), or great value of Rayleigh number, the renewal air induced by the wall heating is partially aspirated from the top and partially from the base of the column. The limit between these two classes is expressed with the form $R/H \sim (Ra)^n$ (Fig. 2).

The boundary layer model allows us to predict this limit. Indeed the thickness of the boundary layer on the cylindrical wall increases with the height and with the decrease of the wall heat flux. When the thickness at the top reaches the radius of the cylinder, the fresh air cannot penetrate. The results on a vertical plate give the order of magnitude of the thickness [9]:

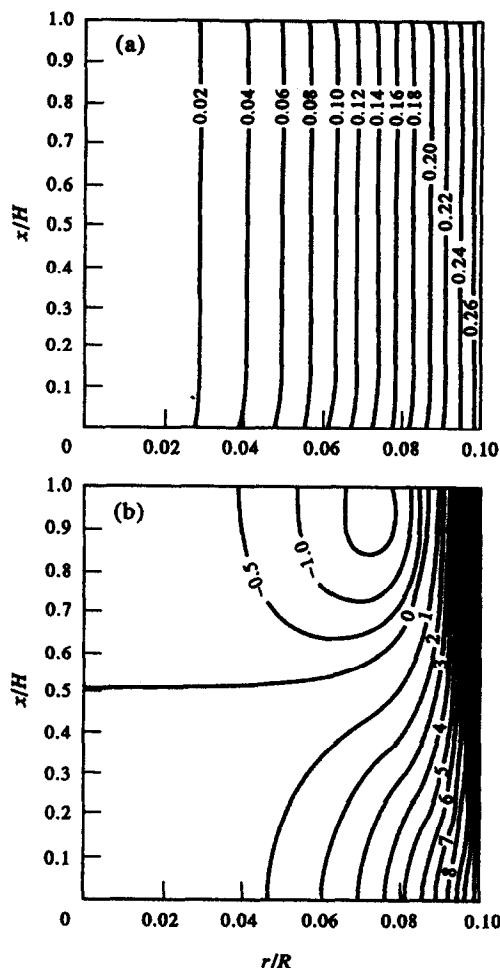


FIG. 1. Natural convection in an open vertical cylinder of porous medium heated with a constant lateral flux. Numerical results of Thibaud [10]. The streamlines correspond to: (a) $Ra = 300$; $R/H = 0.1$; (b) $Ra = 2000$; $R/H = 1$.

$$\delta/x = 4.8(Ra_x)^{-1/3}.$$

Combined with

$$Ra_x = Ra \frac{x^2}{H^2}; \quad x = H; \quad \delta = R$$

this leads to the form $R/H = 4.8Ra^{-1/3}$. Expressed in logarithmic coordinates, this straight line is close to the numerical result of Thibaud (Fig. 2).

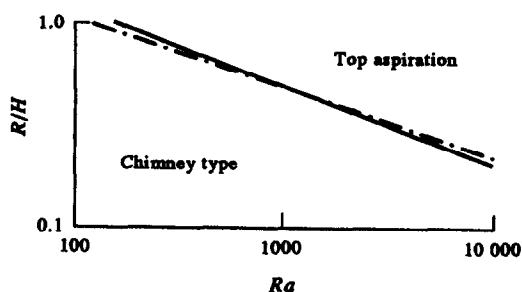


FIG. 2. Limit line between the two flowing types. — Results of Thibaud. - - - Straight line deduced from boundary layer hypothesis.

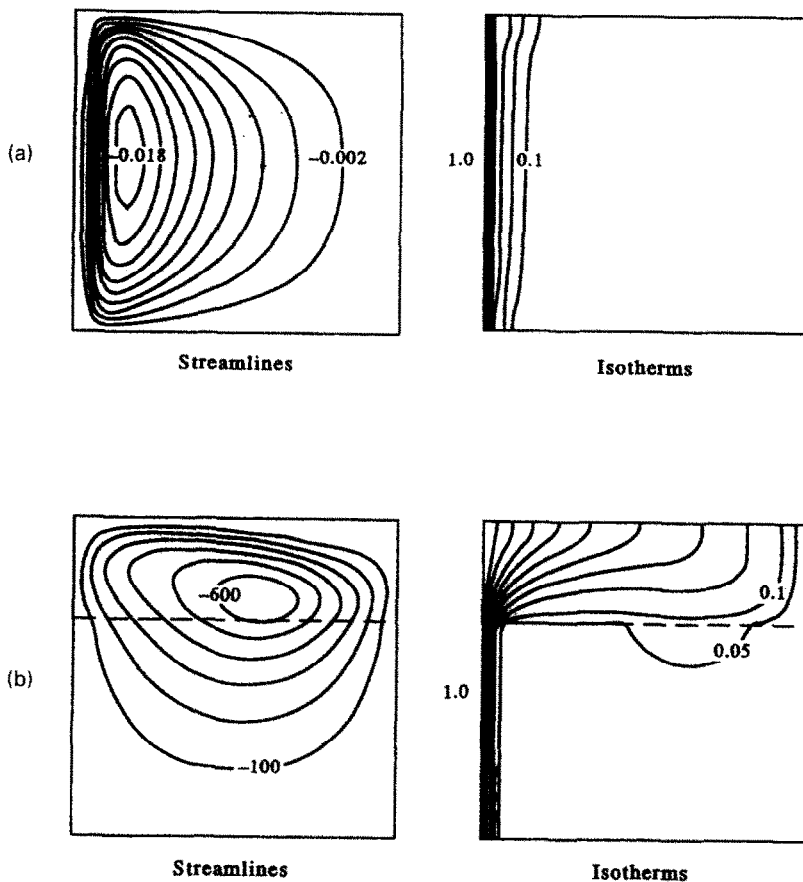


FIG. 3. Simulation of natural convection in a rectangular grain bin. Results of Nguyen [11]. Streamlines and isotherms after six weeks of periodic heating. (a) Confined bin; (b) air headspace bin.

Nguyen carried out numerical simulations of natural convection in rectangular two-dimensional grain bins [11]. The model is composed of conservative equations of mass, energy, moisture and the Darcy law with the transient term. The grain moisture is supposed to be that of thermodynamic equilibrium with air. From an initial uniform state, a periodic temperature (step change day/night) is applied to the left wall, while the right wall is kept at a constant level. Nguyen considers two classes of rectangular bins: the first one perfectly confined (i) and the second one with a headspace of air (ii). For the latter, the coupling with natural convection in the airspace is carried out. The comparison between these two configurations is instructive (Fig. 3).

(i) "At first the heat transfer mainly results in pure conduction as indicated [Fig. 3(a)] in which the isotherms are nearly parallel to the left hand wall. As convection becomes more important, the core region of high intensity flow near the hot wall starts to move slowly towards the centre of the enclosure."

(ii) "The flow reaches steady state very quickly after the beginning of each heating period and is observed to change very little with time. . . . The temperature gradient which exists along the lower part

of the hot wall only produces conduction. After six weeks of alternate heating, the temperature of the grain bulk remains unchanged. . . ."

We keep in mind that periodic heating creates a wall hot layer mainly determined by conduction and the geometry imposes the paths for fresh air supply. Moisture transferred from the grain to the hot air is carried by the convection current to the cooler region.

Smith and Sokhansanj propose a "moisture transport model caused by natural convection in grain stores" [12]. They consider a quasi-equilibrium between air and grain and impose a sinusoid variation of wall temperature over one year. A numerical resolution is carried out in a cylindrical bin.

Few experimental studies are available about this subject. Thus we can quote the paper of Gough [13] which related three years of measurements in two semi-underground hermetic concrete silos filled with maize stored in an equatorial climate. "Considerable moisture content increases occurred at the maize top surface, mainly due to air convection currents at night when the silo structure above ground was cool compared with the grain interior." The temperature and moisture recordings show that the layer affected by

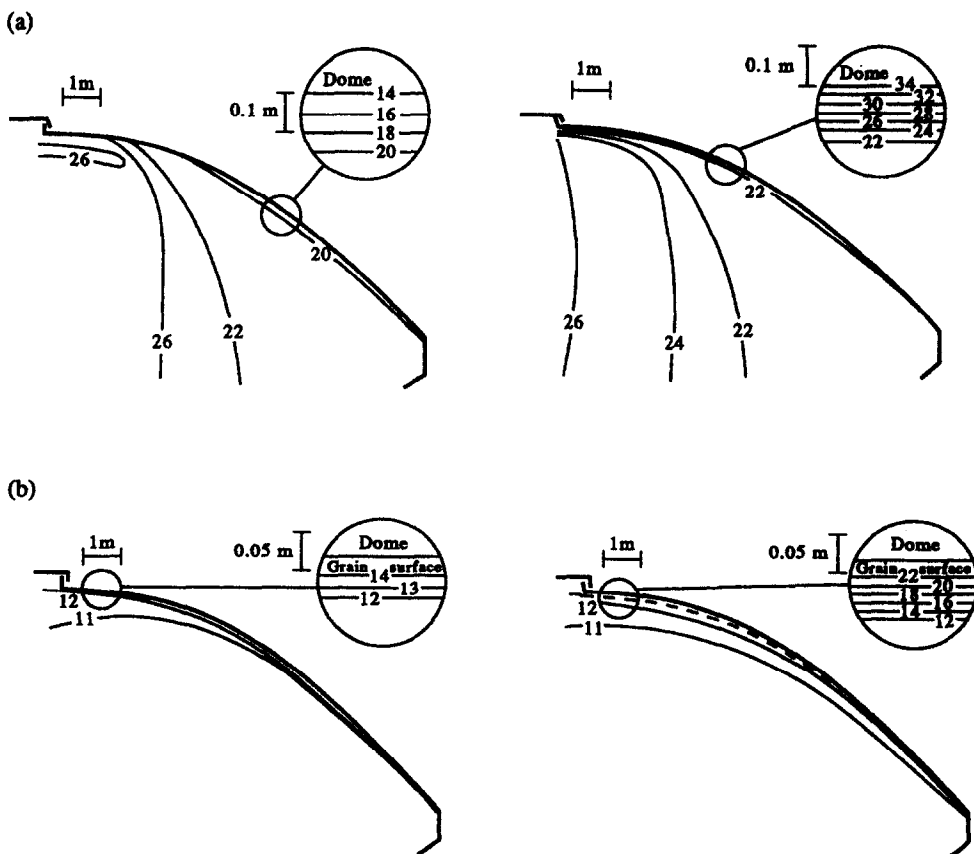


FIG. 4. Experimental results of Gough [13] on a grain storage in a hermetic semi-underground silo. (a) Temperature (°C) in the early morning (7.00 h) (left) and late afternoon (16.00 h) (right) during clear weather (sun exposed faces); (b) moisture content (%) in a silo after 5 months (left) and 7.5 months (right) of storage.

the daily variations is thin (conductive layer), but it creates a natural convection current in the bulk which transports the moisture (Fig. 4). Here we point out the fact that daily weather variations explain the moisture displacement in a silo.

To enlighten the convective problem in a silo, it is convenient to establish the scaling from dimensionless equations.

3. DIMENSIONLESS EQUATIONS OF THE MODEL

We consider an axisymmetrical problem in a cylindrical silo. The following assumptions are used:

- The porous medium is homogeneous and isotropic.
- The physical properties are constant except for the density in the buoyancy term, which is given by the Boussinesq approximation.
- The fluid and the porous medium are in local temperature equilibrium.
- Darcy's law is valid with a transient term.

The conservative and momentum equations are expressed as:

$$\nabla \cdot \mathbf{V} = 0 \quad (\text{mass}) \quad (1)$$

$$\rho_o \left(\varepsilon \frac{\partial Y}{\partial t} + \mathbf{V} \cdot \nabla Y \right) = -(1-\varepsilon) \rho_p \frac{\partial X}{\partial t} \quad (\text{moisture}) \quad (2)$$

$$\frac{\rho_o}{\varepsilon} \frac{\partial \mathbf{V}}{\partial t} = -\nabla P + \rho \mathbf{G} - \frac{\mu}{K} \mathbf{V} \quad (\text{momentum}) \quad (3)$$

$$\rho = \rho_o (1 - \beta(T - T_o) - \beta_v(Y - Y_o)) \quad (\text{air density}) \quad (4)$$

$$(\rho C)^* \frac{\partial T}{\partial t} + \rho_o (C_a + Y C_v) \mathbf{V} \cdot \nabla T = \lambda^* \nabla \cdot \nabla T + L^* (1 - \varepsilon) \rho_p \frac{\partial X}{\partial t} \quad (\text{enthalpy}) \quad (5)$$

where $(\rho C)^* = \varepsilon \rho_o (C_a + Y C_v) + (1 - \varepsilon) \rho_p (C_p + X C_w)$.

Considering the low temperature level, it is convenient to consider the factor $C_a + Y C_v$ as a constant, C_g . No information is available on the drying rate under these conditions. It must express a drying or imbibition phenomenon around the equilibrium. One can expect a linear dependence as:

$$(1 - \varepsilon) \rho_p \dot{X} = -\rho \zeta k (X - X_{eq}) \quad (6)$$

where $X_{eq}(T, Y)$ is given with experimental

correlations [13], and k is the equivalent mass exchange coefficient. Besides, when we consider the slow evolution in air temperature and moisture, it seems right to write:

$$X = X_{\text{eq}}(T, Y). \quad (7)$$

Thus

$$\dot{X} = \frac{\partial X_{\text{eq}}}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial X_{\text{eq}}}{\partial Y} \frac{\partial Y}{\partial t}.$$

Some experimental data should be necessary to specify this point.

The correct form of dimensionless equations is tied to physical adequate reference parameters. The leading idea is that the problem is a conductive one, perturbed by the natural convection. As the periodic flux at the wall governs the exchanges and the natural convection, the first parameters to consider are the 'excitatory period' τ_{ex} and a conductive length δ_{cond} such as:

$$\delta_{\text{cond}}^2 = \tau_{\text{ex}} \cdot \alpha^*. \quad (8)$$

This length is obtained in a classic manner with a balance between the transient term of energy equation and the conductive one:

$$(\rho C)^* \frac{\partial T}{\partial t} \sim \lambda^* \nabla \cdot \nabla T.$$

For this axisymmetric problem

$$\nabla \cdot \nabla T = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial x^2}$$

gives

$$\nabla \cdot \nabla T \sim \frac{\Delta T}{\delta_{\text{cond}}^2}; \quad \left(\frac{\partial}{\partial x} \ll \frac{\partial}{\partial r} \right)$$

and thus

$$\frac{(\rho C)^*}{\tau_{\text{ex}}} \sim \frac{\lambda^*}{\delta_{\text{cond}}^2}$$

which fixes the value of δ_{cond}^2 as

$$\tau_{\text{ex}} \cdot \alpha^*; \quad \left(\alpha^* = \frac{\lambda^*}{(\rho C)^*} \right).$$

A conductive celerity is expressed as:

$$V_{\text{cond}} = \frac{\delta_{\text{cond}}}{\tau_{\text{ex}}}. \quad (9)$$

The silo dimensions give two other geometrical scales: R, H .

As the heat flux density at the wall is imposed $q(t) = \lambda^*(\partial T/\partial r)_w$, a reference flux, q_{ref} , is given and a temperature scale is:

$$\Delta T_{\text{cond}} = q_{\text{ref}} \frac{\delta_{\text{cond}}}{\lambda^*}. \quad (10)$$

A scale for natural convection velocity is deduced from equations (3) and (4) when we consider that, in

the Darcy law, buoyancy and viscosity terms are of the same order of magnitude:

$$\rho_o \beta \Delta T_{\text{cond}} g \sim \frac{\mu}{K} V.$$

Thus

$$V_{\text{n.c.}} = \rho_o g \beta \frac{K}{\mu} \Delta T_{\text{cond}}. \quad (11)$$

The air moisture scale is defined with adiabatic saturation conditions from ambient air:

$$\Delta Y = Y_{\text{sat}}(T_o + \Delta T_{\text{cond}}) - Y_o. \quad (12)$$

The scale of grain water content is deduced from the equilibrium function (sorption or desorption law) $X_{\text{eq}}(T, Y)$:

$$\Delta X = \left(\frac{\partial X_{\text{eq}}}{\partial T} \right)_o \Delta T_{\text{cond}} + \left(\frac{\partial X_{\text{eq}}}{\partial Y} \right)_o \Delta Y. \quad (13)$$

The scale of pressure following natural convection is obtained when we write that the pressure gradient is of the same order of magnitude as the viscosity term (equation (3)):

$$\Delta P = \frac{\mu}{K} V_{\text{n.c.}} \delta_{\text{cond}}. \quad (14)$$

With these scales, the convenient dimensionless variables and physical functions are:

$$t_+ = \frac{t}{\tau_{\text{ex}}}; \quad r_+ = \frac{r}{\delta_{\text{cond}}}; \quad x_+ = \frac{x}{H}; \quad T_+ = \frac{T - T_o}{\Delta T_{\text{cond}}};$$

$$u_+ = \frac{u}{V_{\text{n.c.}}}; \quad v_+ = \frac{v}{V_{\text{n.c.}}}; \quad V_+ = \frac{V}{V_{\text{n.c.}}};$$

$$X_+ = \frac{X - X_{\text{in}}}{\Delta X}; \quad Y_+ = \frac{Y - Y_o}{\Delta Y}; \quad P_+ = \frac{P - P_o}{\Delta P}.$$

The system of equations (1)–(5) becomes:

$$\nabla_+ \cdot \mathbf{V}_+ = 0 \quad (15)$$

$$\frac{V_{\text{cond}}}{V_{\text{n.c.}}} \frac{\partial Y_+}{\partial t_+} + \frac{\mathbf{V}_+ \cdot \nabla_+ Y_+}{\varepsilon} = - \frac{(1-\varepsilon)}{\varepsilon} \times \frac{\rho_p}{\rho_o} \frac{\Delta X}{\Delta Y} \frac{V_{\text{cond}}}{V_{\text{n.c.}}} \frac{\partial X_+}{\partial t_+} \quad (16)$$

$$\frac{V_{\text{cond}}}{V_{\text{n.c.}}} \frac{1}{\varepsilon} \frac{\partial \mathbf{V}_+}{\partial t_+} = - \frac{\mu \delta_{\text{cond}}}{\rho_o K V_{\text{n.c.}}} \times \left[\nabla_+ P_+ + \mathbf{V}_+ + \left(T_+ + \frac{\beta_o \Delta Y}{\beta \Delta T} Y_+ \right) \mathbf{i} \right] \quad (17)$$

$$\frac{\partial T_+}{\partial t_+} + \frac{\rho_o C_g}{\lambda^*} V_{\text{n.c.}} \delta_{\text{cond}} \mathbf{V}_+ \cdot \nabla_+ T_+ = \nabla_+^2 T_+ + \frac{L^*(1-\varepsilon) \rho_p \Delta X}{(\rho C)^* \Delta T_{\text{cond}}} \frac{\partial X_+}{\partial t_+} \quad (18)$$

where:

• in the operator ∇_+ , the factor δ_{cond}/H appears before $\partial/\partial x_+$,

- \mathbf{i} is the unit vector in the direction of \mathbf{G} ,
- the static relation $-\nabla P_0 + \rho_0 \mathbf{G} = 0$ is used for equation (17).

Some dimensionless numbers appear in this system. Now we are going to discuss the physical interpretation and the order of magnitude of the terms.

$$\frac{V_{\text{cond}}}{V_{\text{n.c.}}}$$

is the ratio of the conductive celerity on the natural convection velocity.

$$\mathcal{V} = \frac{1 - \varepsilon}{\varepsilon} \frac{\rho_p}{\rho_o} \frac{\Delta X}{\Delta Y} \frac{V_{\text{cond}}}{V_{\text{n.c.}}}$$

is the coefficient of the source term of humidity in equation (16).

$$V_{\text{d.f.}} = \frac{\varepsilon}{1 - \varepsilon} \frac{\rho_o \Delta Y}{\rho_p \Delta X} V_{\text{n.c.}}$$

is the celerity of a drying front in a fixed bed submitted to the velocity $V_{\text{n.c.}}$ [14], thus \mathcal{V} is the ratio of two celerities:

$$\mathcal{V} = \frac{V_{\text{cond}}}{V_{\text{d.f.}}}$$

The coefficient which appears in the momentum equation (17) can be transformed into:

$$\frac{\delta_{\text{cond}}^2}{K} \frac{1}{Re}$$

where $Re = V_{\text{n.c.}} \delta_{\text{cond}} / \nu$ is the Reynolds number of the flow in the conductive layer. $\delta_{\text{cond}}^2 / K$ scales the medium permeability with the thickness of the conductive layer.

$$N = \frac{\beta_v \Delta Y}{\beta \Delta T}$$

is the classical number which scales natural mass convection before the thermic one.

The numbers which appear in the energy equation (18) are:

$$Ra = \frac{\rho_o C_g V_{\text{n.c.}} \delta_{\text{cond}}}{\lambda^*}$$

which is the Darcy modified Rayleigh number with the conductive length δ_{cond} ;

$$Q = L^* \frac{(1 - \varepsilon) \rho_p}{(\rho C)^*} \frac{\Delta X}{\Delta T_{\text{cond}}}$$

which is the ratio of the latent heat for drying on the sensible heat provided at the wall.

The dimensionless system of equations, where we omit the subscript +, becomes:

$$\nabla \cdot \mathbf{V} = 0 \quad (19)$$

$$\frac{V_{\text{cond}}}{V_{\text{n.c.}}} \frac{\partial Y}{\partial t} + \frac{\mathbf{V}}{\varepsilon} \cdot \nabla Y = -\mathcal{V} \frac{\partial X}{\partial t} \quad (20)$$

$$\frac{V_{\text{cond}}}{V_{\text{n.c.}}} \frac{1}{\varepsilon} \frac{\partial \mathbf{V}}{\partial t} = -\frac{\delta_{\text{cond}}^2}{K} \frac{1}{Re} [\nabla P + \mathbf{V} + (T + NY)\mathbf{i}] \quad (21)$$

$$\frac{\partial T}{\partial t} + Ra \mathbf{V} \cdot \nabla T = \nabla^2 T + Q \frac{\partial X}{\partial t} \quad (22)$$

Let us consider the magnitude of these dimensionless numbers which appear as coefficients. We assume a representative maize silo submitted to the daily atmospheric variations. The magnitude of the heat flux depends on meteorological data and wall constitution. A 'near wall temperature' is evaluated from experimental recordings [12, 14] as $\Delta T_{\text{cond}} = 20^\circ\text{C}$. The medium characteristics are chosen as [15]:

$$K = 3.5 \times 10^{-8} \text{ m}^2; \quad \lambda^* = 0.15 \text{ W m}^{-1} \text{ K}^{-1};$$

$$\alpha^* = 8.8 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}; \quad (1 - \varepsilon) \rho_p = 815 \text{ kg m}^{-3};$$

$$\rho_o = 1.2 \text{ kg m}^{-3}; \quad \varepsilon = 0.4; \quad \mu = 1.8 \times 10^{-5} \text{ Pa s};$$

$$\beta = 3.3 \times 10^{-3} \text{ K}^{-1}; \quad \beta_v = 0.6;$$

$$C_g = 10^3 \text{ J kg}^{-1} \text{ K}^{-1}; \quad \Delta X = 2 \times 10^{-2};$$

$$\Delta Y = 7 \times 10^{-3}; \quad L^* = 2.5 \times 10^6 \text{ J kg}^{-1}.$$

With $\tau_{\text{ex}} = 24$ h, it follows that:

$$\delta_{\text{cond}} = 8.7 \times 10^{-2} \text{ m}; \quad V_{\text{cond}} = 10^{-6} \text{ m s}^{-1};$$

$$V_{\text{n.c.}} = 1.5 \times 10^{-3} \text{ m s}^{-1}; \quad \frac{V_{\text{cond}}}{V_{\text{n.c.}}} = 0.67 \times 10^{-3};$$

$$N = 0.06; \quad \mathcal{V} = 3.2; \quad Re = 8.7;$$

$$\frac{\delta_{\text{cond}}^2}{K} = 2.1 \times 10^5; \quad Ra = 1.04; \quad Q = 1.2.$$

We can make some conclusions about the equations:

- the nonstationary terms $\partial Y / \partial t$, $\partial V / \partial t$ are negligible,
- the thermic natural convection is predominant over the mass one.

The convenient system becomes:

$$\nabla \cdot \mathbf{V} = 0 \quad (23)$$

$$\frac{\mathbf{V}}{\varepsilon} \cdot \nabla Y = -\mathcal{V} \frac{\partial X}{\partial t} \quad (24)$$

$$\nabla P + \mathbf{V} + T\mathbf{i} = 0 \quad (25)$$

$$\frac{\partial T}{\partial t} + Ra \mathbf{V} \cdot \nabla T = \nabla^2 T + Q \frac{\partial X}{\partial t} \quad (26)$$

and an adequate form for the drying rate (equation (6) or (7)). With the chosen numerical values, the conductive and convective terms of energy equation (26) have the same order of magnitude. As the boundary condition is a heat flux at the vertical wall, one can expect mainly a conductive transfer normal to the wall and a convective transfer along the wall. This fact is also seen with the condition $V_{\text{cond}} / V_{\text{n.c.}} \ll 1$. At each moment the convective boundary layer can be considered as established upon a nonstationary conductive layer.

This transfer model is based on some physical con-

siderations which induce a choice of scales. In order to determine the feature of the transfer phenomenon, we carried out an experimental study in a laboratory with a smaller silo.

4. SIMILITUDE PROBLEM

A cylindrical silo of scale about 1/5 to 1/10 was built ($H = 1.86 \text{ m}$; $R = 0.20 \text{ m}$). Since agricultural grains are not convenient for successive drying and rewetting, we used a porous mineral grain of expanded clay pellets: 6 mm mean diameter, $(1 - \epsilon)\rho_p = 497 \text{ kg m}^{-3}$; $K = 3.6 \times 10^{-8} \text{ m}^2$; $\lambda^*(\text{dry grain}) = 0.138 \text{ W m}^{-1} \text{ K}^{-1}$; $\lambda^*(\text{wetted to } 5\% \text{ d.b.}) = 0.153 \text{ W m}^{-1} \text{ K}^{-1}$; $C^* = 836 \text{ J kg}^{-1} \text{ K}^{-1}$.

The true similitude would be to impose the same values for the dimensionless numbers \mathcal{V} , Ra , Q . The constitutive main factors are related to the conductive transfer determined with the boundary conditions τ_{ex} , q_{ref} or ΔT_{cond} . The choice of these factors is guided by the necessity to simulate a conductive layer at the wall whose thickness should be smaller than the radius. The fixed value $\tau_{ex} = 2 \text{ h}$ leads to $\delta_{cond} = 5.4 \times 10^{-2} \text{ m}$ compatible with the value of R and convenient for the sensor equipment.

As the periodic heat flux at the wall cannot be obtained with a zero mean value, a square pulse heat flux (180 W m^{-2}) is applied during a small part (1/8) of the period (2 h). The mean flux (22.5 W m^{-2}) remains a secondary factor. As we shall see later on, the applied flux does not diffuse entirely towards the interior.

The temperature scale ΔT_{cond} (45 K) can be estimated for an intermediate flux between the maximum and the mean value. In this medium, with the physical data, the characteristic numbers are:

$$V_{n.c.} = 2.8 \times 10^{-3} \text{ m s}^{-1}; \quad V_{cond} = 7.0 \times 10^{-6} \text{ m s}^{-1};$$

$$V_{d.f.} = 3.1 \times 10^{-6} \text{ m s}^{-1} (\Delta X = 5 \times 10^{-3});$$

$$\mathcal{V} = 2.25; \quad Ra = 1.15; \quad Q = 3.32.$$

As for a real corn silo, the values of the numbers \mathcal{V} , Ra , Q are close to unity.

5. EXPERIMENTAL APPARATUS

The heat flux is furnished by an electric annular film at the wall of the cylinder, constituted of 8 elements with individual regulations (Fig. 5). Around the cylinder a rockwool thickness, another metallic cylinder and another insulation establish a good boundary condition for the granular medium. Indeed the external metallic cylinder is surrounded by a roll of electric tape and the regulation attempts to assure the same temperature at the wall on the two cylinders. This process is efficient for a stationary case but less so when the heat flux moves. Thermal capacities of walls and insulations disturb the regulation. Consequently the real flux injected in the granular medium is deduced from temperature measurements (Fig. 6).

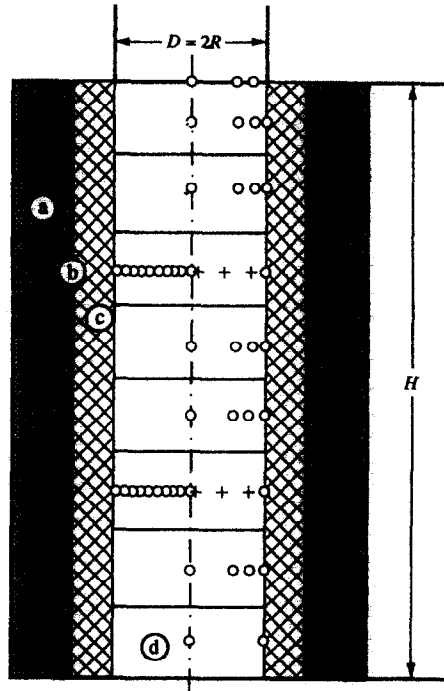


FIG. 5. Silo constitution and localization of measuring instruments. $H = 1.86 \text{ m}$; $D = 0.4 \text{ m}$. (a) rockwool insulation; (b) metal wall with electric coil rolling around; (c) rockwool insulation; (d) annular electric film heater which constitutes the inner cylinder full of expanded clay pellets. (o) Thermocouple inside pellet. (+) air moisture sensor.

The temperature field is investigated with 38 thermocouples introduced into individual pellets. The air moisture is measured by 6 sensors.

Two particular sections of the silo are plentifully equipped (Fig. 5). A microcomputer associated with a regulation module assures the acquisition and treatment of the data. The granular medium is poured into the silo with a uniform water content of 5%.

6. GLOBAL BALANCES FOR MEASUREMENT INTERPRETATION

The evolution of the temperature field $T(r, x, t)$ in the silo is the key to reach natural convection and

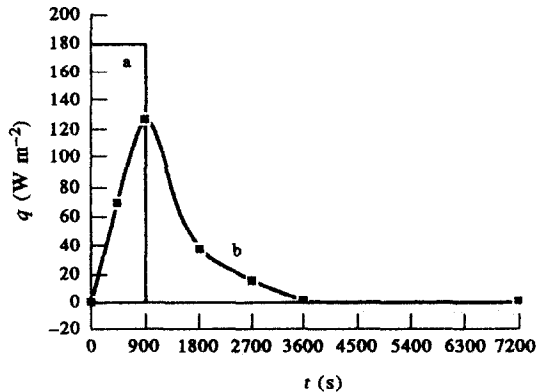


FIG. 6. Periodic thermal flux at the wall of the silo. (a) Applied to the wall; (b) injected through granular medium.

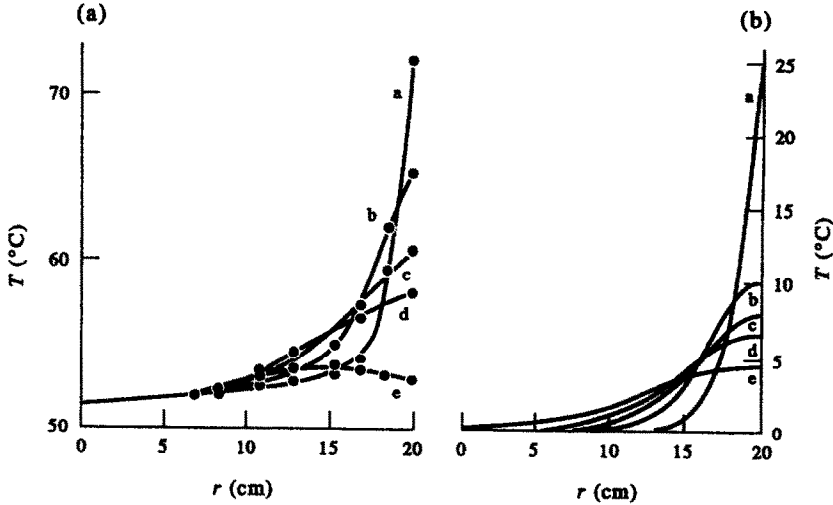


FIG. 7. (a) Damping of the temperature profile after the square heat flux. $T(r, x, t)$; $x = 0.58$ m; $t = \tau_{ex}(n + (1/p))$; $n = 120$; $\tau_{ex} = 2$ h; $p = \frac{1}{8}$ (a); $\frac{1}{4}$ (b); $\frac{3}{8}$ (c); $\frac{1}{2}$ (d); 1 (e). (b) Temperature profile in a semi-infinite medium after a square heat flux. $q = 180$ W m⁻², $t = \tau_{ex}/p$ with same values for p as in (a).

drying phenomenon. As the flux at the wall is periodic we can note $t = \tau_{ex}(n + (1/p))$, n and p integers. The conductive layer associated with the periodic flux agrees with the previous theory (Fig. 7(a)). We can compare the measured field to the temperature profiles $T(r, t)$ resulting from an analytical solution of a similar problem: a square heat flux at the wall of a semi-infinite medium (Fig. 7(b)) [16].

The natural convection velocity is linked to the temperature field and can be extracted through a global balance in the silo. Following a previous physical description, we first focus on the low part of the silo, where the effect of the recirculation is negligible, i.e. $v \ll u$. Using the system (23)–(26) written in cylindrical coordinates, after elimination of P , equation (25) becomes:

$$\frac{\partial u}{\partial r} - \frac{\delta_{cond}}{H} \frac{\partial v}{\partial x} - \frac{\partial T}{\partial r} = 0. \quad (27)$$

The cylindrical conductive layer phenomenon allows:

$$\frac{\partial v}{\partial x} \ll \frac{\partial u}{\partial r} \quad (28)$$

and thus equation (27) is integrated as

$$u = T + a(x). \quad (29)$$

Equations (23), (24), (26) are expressed in cylindrical coordinates as

$$\frac{\delta_{cond}}{H} \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial rv}{\partial r} = 0 \quad (30)$$

$$\frac{\delta_{cond}}{H} u \frac{\partial Y}{\partial x} + v \frac{\partial Y}{\partial r} = -\epsilon \gamma \frac{\partial X}{\partial t} \quad (31)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + Ra \left(\frac{\delta_{cond}}{H} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) &= \frac{\delta_{cond}^2}{H^2} \frac{\partial^2 T}{\partial r^2} \\ &+ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + Q \frac{\partial X}{\partial t}. \end{aligned} \quad (32)$$

The boundary conditions are:

$$v(r_w, x) = 0; \quad v(0, x) = 0$$

$$T(r, 0) = T_o; \quad Y(r, 0) = Y_o$$

$$\left(\frac{\partial T}{\partial r} \right)_w = q(t).$$

We note that:

$$u_o S = \int_0^{r_w} u(r, 0) 2\pi r dr.$$

The global heat and mass balance of the granular medium are obtained with integration over the whole section and over a part of the height:

$$\int_0^x \int_0^{r_w} \dots 2\pi r dr dx.$$

Equation (30), taking into account (29), gives:

$$a(x) = u_o - \langle T \rangle(x)$$

where $\langle T \rangle$ is the mean temperature in the section:

$$S \langle T \rangle = \int_0^{r_w} T dS; \quad S = \pi r_w^2. \quad (33)$$

It follows that:

$$u = u_o + T - \langle T \rangle.$$

Equation (31) gives:

$$\frac{\delta_{\text{cond}}}{H} \left[\int_0^{r_w} uY2\pi r dr - u_o Y_o S \right] = -\varepsilon \mathcal{V} S \frac{d}{dt} \int_0^x \langle X \rangle dx \quad (34)$$

where $\langle X \rangle$ is the mean water content in the section. Equation (34) expresses the decrease of water content with the air moisture convection.

Equation (32), taking (30) into account, gives:

$$S \frac{\partial}{\partial t} \int_0^x \langle T \rangle dx + Ra \frac{\delta_{\text{cond}}}{H} \left[\int_0^{r_w} uT dS - u_o T_o S \right] = \frac{\delta_{\text{cond}}^2}{H^2} S \left(\frac{\partial \langle T \rangle}{\partial x}(x) - \frac{\partial \langle T \rangle}{\partial x}(o) \right) + \int_0^x \mathbf{q} \cdot 2\pi r_w dx + QS \frac{\partial}{\partial t} \int_0^x \langle X \rangle dx. \quad (35)$$

Equation (35) expresses the relation between the convective heat flux, the cumulative sensible heat, the conductive flux through the wall and the limit sections, and the latent heat flux of evaporation.

The system (33)–(35) allows us to deduce the natural convective velocity u_o . We remember that the validity of equation (33) is related to the condition $(\partial v/\partial x) \ll (\partial u/\partial r)$, which means that the influence of a top aspiration is negligible.

The convective terms of equations (34), (35) become:

$$\int_0^x uY dS = (u_o \langle Y \rangle + \langle TY \rangle - \langle T \rangle \langle Y \rangle) S$$

$$\int_0^{r_w} uT dS = (u_o \langle T \rangle + \langle T^2 \rangle - \langle T \rangle^2) S.$$

As expected, the quadratic terms $\langle TY \rangle - \langle T \rangle \langle Y \rangle$ and $\langle T^2 \rangle - \langle T \rangle^2$ are negligible compared to the others ($\sim 10^{-3}$).

Equations (34), (35) become:

$$\frac{\delta_{\text{cond}}}{H} u_o (\langle Y \rangle - Y_o) = -\varepsilon \mathcal{V} \frac{\partial}{\partial t} \int_0^x \langle X \rangle dx \quad (36)$$

$$Ra \frac{\delta_{\text{cond}}}{H} u_o (\langle T \rangle - T_o) = -\frac{\partial}{\partial t} \int_0^x \langle T \rangle dx + \frac{\delta_{\text{cond}}^2}{H^2} \left(\frac{\partial \langle T \rangle}{\partial x}(x) - \frac{\partial \langle T \rangle}{\partial x}(o) \right) + \frac{1}{S} \int_0^x q(t) 2\pi r_w dx + Q \frac{\partial}{\partial t} \int_0^x \langle X \rangle dx. \quad (37)$$

The elimination of the drying rate

$$\frac{\partial}{\partial t} \int_0^x \langle X \rangle dx$$

between (36), (37) gives the expression for u_o :

$$\frac{\delta_{\text{cond}}}{H} \left[Ra(\langle T \rangle - T_o) + \frac{Q}{\varepsilon \mathcal{V}} (\langle Y \rangle - Y_o) \right] u_o = -\frac{\partial}{\partial t} \int_0^x \langle T \rangle dx + \frac{\delta_{\text{cond}}^2}{H^2} \left(\frac{\partial \langle T \rangle}{\partial x}(x) - \frac{\partial \langle T \rangle}{\partial x}(o) \right) + \frac{1}{S} \int_0^x q(t) 2\pi r_w dx. \quad (38)$$

7. THE TOP ASPIRATION PHENOMENON

The recording of grain temperatures and air moisture allowed us to evaluate each term of equation (38). We chose two particular periods: during the drying (period 66) and at the end of the experiment (period 132) when all the grain has become dry ($\langle Y \rangle = Y_o$).

Without a top aspiration, the u_o value does not depend upon the integration limit for x . Figures 8(a) and 8(b) show that a top aspiration may concern the upper third of the silo during the first half of the period. We note that the convective flowrate is more

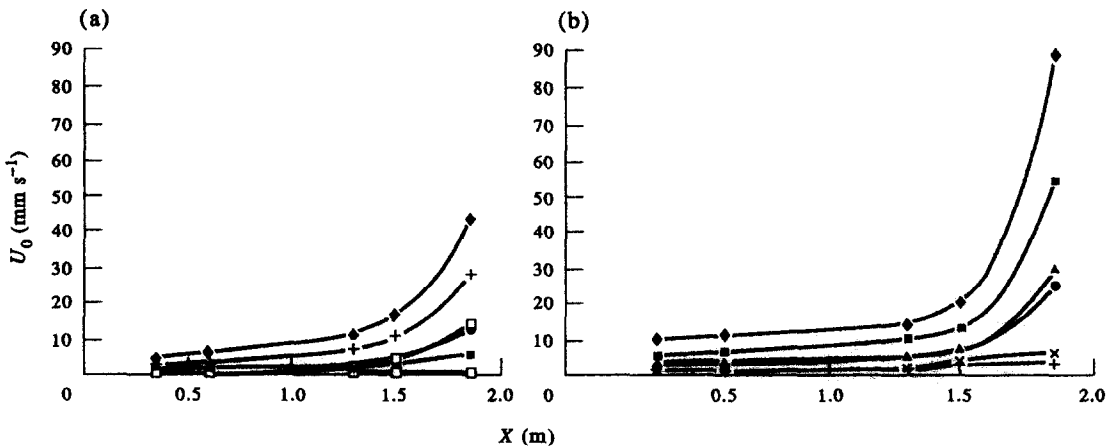


FIG. 8. Natural convection velocity u_o , obtained from equation (38). (a) Period 66 (during the drying). (b) Period 132 (after the drying). t (s): \triangle 450; \blacklozenge 900; \square 1800; \blacktriangledown 2700; \times 3600; $+$ 7200.

Table 1. Enthalpy fluxes which constitute the energy equation (39) (W). Period 66 (during the drying)

$r(s)$	Enthalpy flux at the silo top $\phi(H)$	Enthalpy flux at the silo bottom $\phi(o)$	Conductive loss $\lambda^*(\partial\langle T\rangle/\partial x)_o^H S$	Heat flux at the wall $2\pi R\lambda^* \int_0^H (\partial T/\partial r)_R dx$	Rate of heat storage $(\rho c)*S(\partial/\partial t) \int_0^H \langle T\rangle dx$
450	86	6	-7	150	63
900	87	34	-7	157	97
1800	180	18	-7	143	-26
2700	86	5	-7	62	-26
3600	42	2	-7	21	-26
7200	28	1	-7	8	-26

Table 2. Enthalpy fluxes which constitute the energy equation (39) (W). Period 132 (after the drying)

$r(s)$	Heat flux at the silo top $\phi(H)$	Heat flux at the silo bottom $\phi(o)$	Conductive loss $\lambda^*(\partial\langle T\rangle/\partial x)_o^H S$	Heat flux at the wall $2\pi R\lambda^* \int_0^H (\partial T/\partial r)_R dx$	Rate of heat storage $(\rho c)*S(\partial/\partial t) \int_0^H \langle T\rangle dx$
450	63	10	-16	161	92
900	188	32	-16	296	124
1800	113	17	-16	88	-24
2700	50	7	-16	35	-24
3600	14	3	-16	3	-24
7200	8	2	-16	-2	-24

important when the grain is dry. Indeed, the mainly thermic natural convection is weaker during the drying when a part of the heat flux at the wall is used for evaporation.

The value of the convective velocity u_o (Fig. 8; $x < 1$ m) is suitable for an evaluation of the flow enthalpy in the top section of the silo.

We can eliminate the latent flux of evaporation between equations (34), (35), and integrate the result over the distance $0-H$. The dimensional expression is:

$$\phi(H) = \phi(o) + \lambda^* S \left[\frac{\partial\langle T\rangle}{\partial x}(H) - \frac{\partial\langle T\rangle}{\partial x}(o) \right] + 2\pi R\lambda^* \int_0^H \left(\frac{\partial T}{\partial r} \right)_R dx - (\rho C) * S \frac{d}{dt} \int_0^H \langle T\rangle dx \quad (39)$$

where the enthalpy flux is:

$$\phi = \rho_o \int_0^R (C_a T + Y L^*) u dS.$$

Tables 1 and 2 show the value of the terms of equation (39) for the same periods used for the determination of u_o . When the grain is dry (2), the enthalpy flux reduces to a heat flux.

We note the delay between the admittance of the heat flux through the wall and the convective outlet at the silo top. The heat storage in the 'conductive layer' is the predominant factor.

8. DRYING EVOLUTION IN THE SILO

As the mean heat flux at the wall is not null, the entire mass of grains is affected by the drying. We should consider that the heat flux applied can be Four-

ier decomposed into a series of sinusoid terms and the mean value. Although this problem is not linear as a purely conductive one would be, we can roughly interpret the experimental results as a superposition of solutions.

We examine the drying of all the mass through the recorded temperatures (and moistures) in the middle of the period $T(r, x, t = \tau_{ex}(n+0.5))$. The drying is determined by two factors: the heat flux from the wall and the drying capacity of air ($Y_{sat}(T) - Y$) which issues from the bottom and the top of the silo through natural convection. We note that the drying front mainly moves from the wall towards the axis (Figs. 9 and 10) and secondarily from the bottom towards the

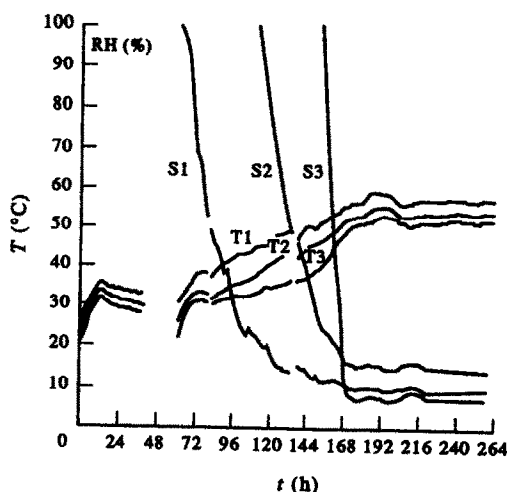


FIG. 9. Drying of the grain: grain temperature (T) and air moisture (S). RH is the relative humidity. $T(r, x, t)$; $x/H = 0.312$; $t = \tau_{ex}(n+0.5)$; $\tau_{ex} = 2$ h. 1: $r/R = 0.92$; 2: $r/R = 0.50$; 3: $r = 0$.

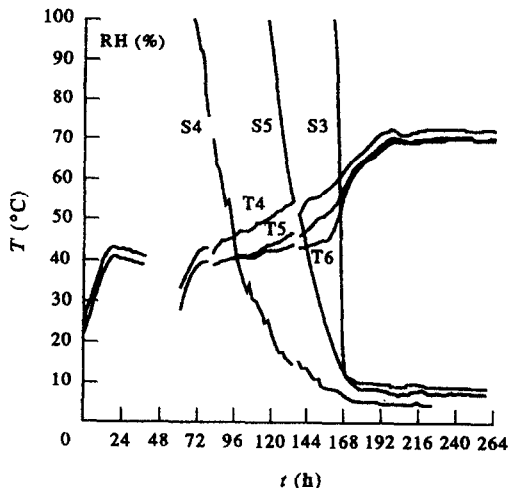


FIG. 10. Drying of the grain: grain temperature (T) and air moisture (S). RH is the relative humidity. $T(r, x, t)$; $x/H = 0.687$; $t = \tau_{ex}(n+0.5)$; $\tau_{ex} = 2$ h. 4: $r/R = 0.92$; 5: $r/R = 0.50$; 6: $r = 0$.

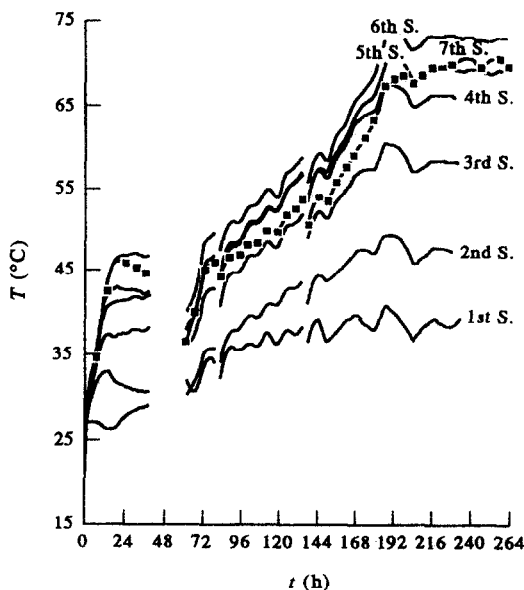


FIG. 12. Drying of the grain: temperature at the wall. $T(r, x, t)$; $r = R$; $t = \tau_{ex}(n+0.5)$; $\tau_{ex} = 2$ h. Positions of the sections: see Fig. 11.

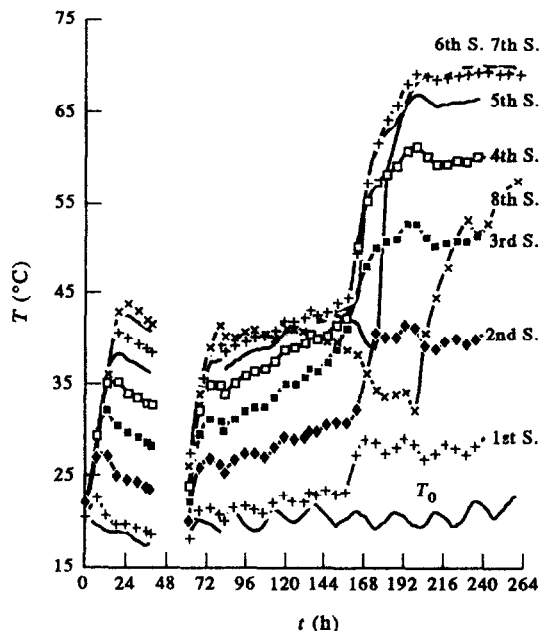


FIG. 11. Drying of the grain: axial temperature. $T(r, x, t)$; $r = 0$; $t = \tau_{ex}(n+0.5)$; $\tau_{ex} = 2$ h. The positions of the sections are (x/H) : 1st S. (0.062); 2nd S. (0.187); 3rd S. (0.312); 4th S. (0.437); 5th S. (0.562); 6th S. (0.687); 7th S. (0.812); 8th S. (0.937); T_0 : ambient temperature.

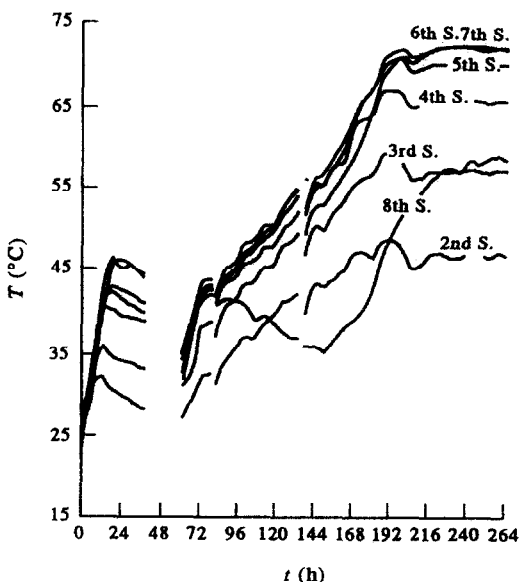


FIG. 13. Temperature profiles along a vertical straight. $T(r, x, t)$; $t = \tau_{ex}(n+0.5)$; $\tau_{ex} = 2$ h; $r = R - 1$ cm. Positions of the sections: see Fig. 11.

top (Fig. 11). However, this front is very particular. The heat flux and the air drying capacity are not associated with the air flow as for the usual convective drying in a static drier. We must consider a front shape in the space which moves in two directions. We can see (Figs. 9–13) that the drying is over at approximately the same time in the whole bulk (hour 192), which means that the natural flowrate induced by the temperature field is not much different from one point to another. We remember that the top aspiration is probably negligible during the second part of the

period and thus the ‘chimney’ type convective phenomenon facilitates a good irrigation of the medium.

We remark that the temperature evolution of the points situated on the 8th section (12 cm under the top) is specious. The initial temperature increase is followed later by a phase of temperature decrease (Figs. 11 and 13). The decreasing temperature means that a part of the evaporative latent flux is taken from the enthalpy flux of air and this air mainly issues from the top space (recirculation).

From the mean heat flux \bar{q} applied at the wall,

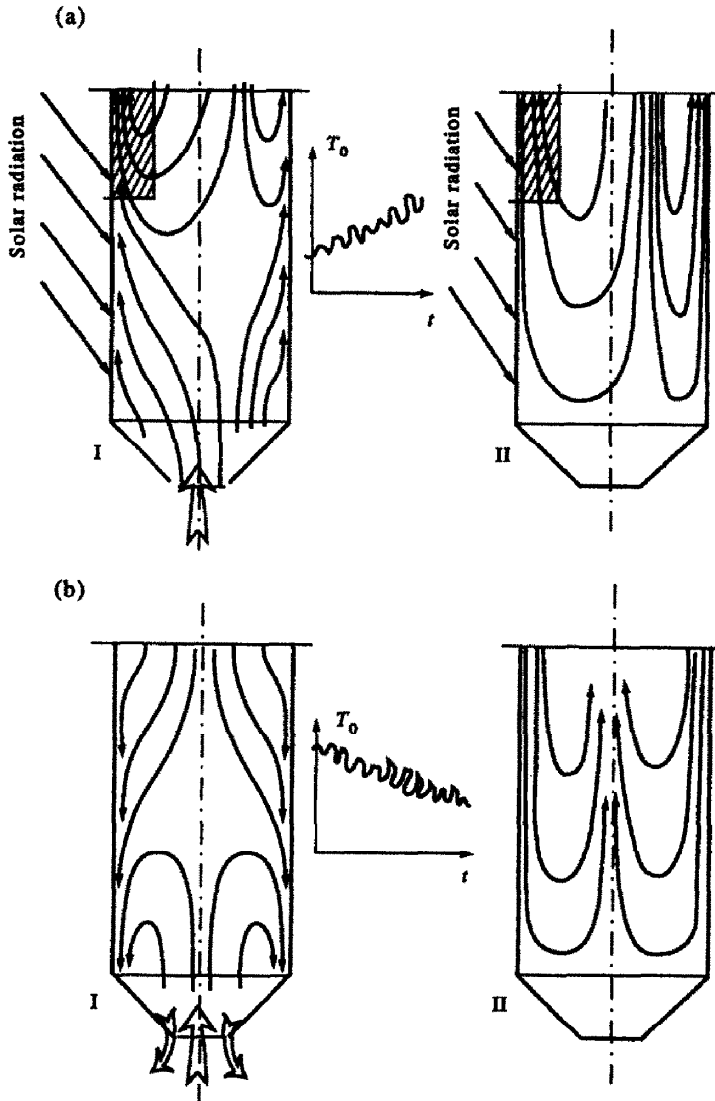


FIG. 14. Sketch of natural convective flow and moisture transfer expected into two types of silos. (I—bottom opened; II—bottom closed). Two seasonal periods are considered: from winter to summer (north latitude) (a); from summer to winter (b). Hatched areas are the foreseeable condensation places.

we can have a good approximation of the moisture evaporation. The natural convection velocity (equation (11)) is obtained from the scales:

$$V_{n.c.} = \rho_o g \beta \frac{K}{\mu} \Delta T_{cond}$$

where

$$\Delta T_{cond} = \frac{\bar{q}R}{\lambda^*}$$

With $g = 24 \text{ W m}^{-2}$ and the given parameters,

$$\Delta T_{cond} = 31^\circ\text{C}; \quad V_{n.c.} = 2.4 \text{ mm s}^{-1}.$$

If we admit a chimney type convective effect, equation (39) can be integrated over a period:

$$\int_0^\tau (\phi(H) - \phi(o)) dt = \int_0^\tau \lambda^* S$$

$$\times \left(\frac{\partial \langle T \rangle}{\partial x}(H) - \frac{\partial \langle T \rangle}{\partial x}(o) \right) dt$$

$$- 2\pi RH \bar{q} \tau - (\rho C)^* S \left[\int_0^H \langle T \rangle dx \right]_o^\tau. \quad (40)$$

The last term of equation (40) is negligible. Indeed, over a period the increase of the sensible heat stored in the grain is less than the latent heat of evaporation. The conductive loss from the silo extremities is roughly constant during the evaporation phase (Table 1). The convective enthalpy flux is only a function of the outlet air temperature:

$$\int_0^\tau (\phi(H) - \phi(o)) dt = \rho_o [C_a \langle T \rangle + L^* \langle Y \rangle]_o^H U_o \tau$$

where

$$\langle Y \rangle(H) = Y_{\text{sat.}}(\langle T \rangle(H)). \quad (41)$$

We supposed that the outlet air was saturated and at a constant temperature. The registered temperature variations over the top surface are nearly constant ($<1^\circ\text{C}$) during the period. The numerical resolution of equations (40), (41) gives $\langle T \rangle(H) = 39^\circ\text{C}$. The moisture flow from the silo top during a drying period is (dimensional form):

$$\rho_o S(\langle Y \rangle(H) - Y_o) u_o \tau$$

which gives 0.101 kg for the test period.

We can compare this value to the mean values of the drying rate. The grain bulk contains 5 kg of water which is evaporated during 84 periods, i.e. a mean value of 60 g per period. If we consider that the rate is less at the beginning (heating of the grain) and at the end of the experiment (partially dry bulk), this evaluation is close to the real value.

9. CONCLUSION

The experimental investigations carried out on a silo maquette are interpreted in regard to dimensional analysis of the mathematical model. The drying of the grain bulk is the result of the natural convective flow which is induced by the temperature field. As the heat flux applied to the wall is periodic but with a mean value different from zero, the natural velocity field follows two configurations.

During the first half period the main part of the heat and mass transfer takes place in a boundary convective layer established on a conductive hot layer. This layer is fed with fresh air from the bottom and from the top through a central aspiration.

During the second half period the stored heat provides the convective flux as a chimney type without top aspiration and the drying leads to a reduction in the differences of water content. The order of magnitude of evaporated water flowing through the upper level can be obtained with a sufficient accuracy.

The transfer mechanisms examined here can give indications for real silos which are somewhat different from this model. Some additional particular features may be considered:

- strictly airtight at the bottom;
- a closed upper airspace;
- the daily alternance from winter to summer and inversely (Fig. 14);
- the exposure of the wall to the sun (the transfer mechanism is probably limited around the most exposed surface);
- the ratio δ_{cond}/R (if the boundary layer thickness is very narrow, the chimney type does not occur);
- the variation of the heat flux along the wall (the

atmospheric factors, solar radiation, air temperature, wind velocity, do not create a uniform heat flux);

- the constitutive shape of the silo roof (the moist air flow out of the bulk grain can condense on the colder walls and be transformed into water drops).

In order to clarify the third point, we note that the period from winter to summer is critical for the condensation in the grain bulk, whereas the period from summer to winter is favorable for the stored grain. A very cold winter is considered as ideal by the silo managers.

REFERENCES

1. A. Bejan and K. R. Khair, Heat and mass transfer by natural convection in a porous medium, *Int. J. Heat Mass Transfer* **28**, 909–918 (1985).
2. J. Y. Jang and W. J. Chang, Buoyancy induced inclined boundary layer flow in a porous medium resulting from combined heat and mass buoyancy effects, *Int. Com. Heat Mass Transfer* **15**, 17–30 (1988).
3. Adnan Yucel, Natural convection heat and mass transfer along a vertical cylinder in a porous medium, *Int. J. Heat Mass Transfer* **33**, 2265–2274 (1990).
4. P. Cheng and W. J. Minkowycz, Free convection about a vertical flat plate embedded in a saturated porous medium with application to heat transfer from a dike, *J. Geophys. Res.* **82**, 2040–2044 (1977).
5. A. Bejan, Natural convection in confined porous media, Chap. 11 in *Convection Heat Transfer*, Wiley, New York (1984).
6. A. Bejan, Natural convection in a vertical cylindrical well filled with porous medium, *Int. J. Heat Mass Transfer* **23**, 726–729 (1980).
7. M. R. Islam and K. Nandakumar, Transient convection in saturated porous layers with internal heat sources, *Int. J. Heat Mass Transfer* **33**, 151–161 (1990).
8. S. Whitaker, Simultaneous heat, mass and momentum transfer in porous media: a theory of drying. In *Advances in Heat Transfer*, Vol. 13, Academic, New York (1977).
9. P. Cheng, Heat transfer in geothermal systems, *Adv. Heat Transfer* **14**, 1–105 (1979).
10. L. Thibaud, Contribution à l'étude de la convection naturelle à l'intérieur d'un cylindre vertical poreux soumis à une densité de flux thermique pariétal constante, Thèse de l'Université de Poitiers, France (1988).
11. T. V. Nguyen, Natural convection effects in stored grains. A simulation study, *Drying Technol.* **5** (4), 541–560 (1987).
12. E. A. Smith and S. Sokhansanj, Moisture transport caused by natural convection in grain stores, *J. Agric. Engng Res.* **47**, 23–24 (1990).
13. M. C. Gough, Physical changes in large scale hermetic grain storage, *J. Agric. Engng Res.* **31**, 55–65 (1985).
14. G. Arnaud and J.-P. Fohr, Slow drying simulation in thick layers of granular products, *Int. J. Heat Mass Transfer* **31**, 2517–2526 (1988).
15. W. Mühlbauer, Untersuchungen über die trocknung von Körnermais unter besonderer berücksichtigung des glüchstrom-trocknungsverfahrens. Arbeitskreises Forschung und Lehre der Max Eyth-gesellschaft zur förderung der Agartechnik Francfort, CNEEMA Translation (1974).
16. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, Oxford University Press, London (1959).